S-WAVE NEUTRON STRENGTH FUNCTION AND THE OPTICAL MODEL WITH VOLUME AND SURFACE ABSORPTION

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Recently Benczo (1966) has obtained the exact analytical solution of the Schrödinger equation with a complex potential of the form

$$V(r) = -\left[\frac{V_0+iW_0}{1+e^{\frac{r-R}{a}}} + (V_1+iW_1)\frac{e^{\frac{r-R}{a}}}{\left(1+e^{\frac{r-R}{a}}\right)^2}\right] \quad \ldots \quad (1)$$

for s-wave neutrons and has given the explicit expression for the S_0-matrix element as

$$S_0(k) = e^{-2ikR}\,\frac{\Gamma(2ika)}{\Gamma(-2ika)}\cdot\frac{AB-C}{D-AE} \quad \ldots \quad (2)$$

where

$$A = \left(\frac{b}{1+b}\right)^{2\lambda}(1+b)^{-2ika}\frac{F\left(\lambda+\mu+ika,\ 1+\lambda-\mu+ika,\ 1+2\lambda\ ;\ \frac{b}{1+b}\right)}{F\left(-\lambda+\mu-ika,\ 1-\lambda-\mu-ika,\ 1-2\lambda\ ;\ \frac{b}{1+b}\right)} \quad \ldots \quad (3a)$$

F denoting hypergeometric functions and k being the wave number,

$$B = \frac{\Gamma(1-2\lambda)}{\Gamma(1-\lambda-\mu+ika)\,\Gamma(-\lambda+\mu+ika)} \quad \ldots \quad (3b)$$

$$C = \frac{\Gamma(1+2\lambda)}{\Gamma(\lambda+\mu+ika)\,\Gamma(1+\lambda-\mu+ika)} \quad \ldots \quad (3c)$$

$$D = \frac{\Gamma(1+2\lambda)}{\Gamma(1+\lambda-\mu-ika)\,\Gamma(\lambda+\mu-ika)} \quad \ldots \quad (3d)$$

$$E = \frac{\Gamma(1-2\lambda)}{\Gamma(-\lambda+\mu-ika)\,\Gamma(1-\lambda-\mu-ika)} \quad \ldots \quad (3e)$$

in which

$$\lambda = \pm \left[1 + \frac{V_0 + iW_0}{En} \right]^{\frac{1}{2}} \cdot ika \qquad \dots \quad (4a)$$

$$\mu = \frac{1}{2} \pm \frac{1}{2} \left[1 + 4k^2 a^2 \frac{V_1 + iW_1}{E_n} \right]^{\frac{1}{2}} \qquad \dots \quad (4b)$$

E_n = neutron energy

In the above potential the first part represents the usual Woods-Saxon potential and the second part corresponds to its derivative We shall utilise the above results for the calculation of the s-wave neutron strength function $\frac{\bar{\Gamma}_n}{D}$ (the ratio of the average neutron width to the average level spacing) and the potential scattering radius R' using the same form of neutron-nucleus potential with $V_1 = 0$.

According to Feshbach, Porter and Weisskopf (1954)

$$\frac{\bar{\Gamma}_n}{D} = \frac{1}{\pi} \, Re(1 - S_0(k)) \qquad \dots \quad (5)$$

$$kR' = \frac{1}{2} \, \mathrm{Im}(1 - S_0(k)) \qquad \dots \quad (6)$$

for $kR' < < 1$

Since $$\{1 - S_0(k)\}_{k \to 0} = -K_0 \left(\frac{dS_0(k)}{dk} \right)_{k=0}$$

the strength function normalised to 1 ev is given by

$$\frac{\bar{\Gamma}_n^0}{D} = -\frac{1}{\pi} k_0 \, \mathrm{Re} \left(\frac{dS_0(k)}{dk} \right)_{k=0} \qquad \dots \quad (7)$$

and $$R' = -\frac{1}{2} \, \mathrm{Im} \left(\frac{ds_0(k)}{dk} \right)_{k=0} \qquad \dots \quad (8)$$

where k_0 is the value of k for 1 ev neutron.

Now

$$\frac{dS_0(k)}{dk} = -2iR \cdot S_0(k) + \left[\frac{d}{dk} \left\{ \frac{\Gamma(2ika)}{\Gamma(-2ika)} \right\} \right] \cdot \frac{\Gamma(-2ika)}{\Gamma(2ika)} \cdot S_0(k)$$

$$+ \left[e^{-2ikR} \cdot \frac{\Gamma(2ika)}{\Gamma(-2ika)} \right] \cdot \left[\frac{(A'B + AB' - C')(D - AE) - (AB - C)(D - A'E - AE')}{(D - AE)^2} \right] \qquad \dots \quad (9)$$

where dash denotes differentiation with respect to k.

$$\text{We note that } B_0 = E_0$$

$$C_0 = D_0$$

and

$$S_0(0) = 1 \qquad \ldots \quad (10)$$

where the suffix zero indicates the value of the corresponding quantity at $k = 0$
Further we have

$$\left.\frac{dB}{dk}\right|_{k=0} = -\left.\frac{dE}{dk}\right|_{k=0} = -iaB_0\Big[\psi(1-\lambda-\mu)+\psi(-\lambda+\mu)\Big] \qquad . \quad (11a)$$

and

$$\left.\frac{dC}{dk}\right|_{k=0} = -\left.\frac{dD}{dk}\right|_{k=0} = -iaC_0\Big[\psi(\lambda+\mu)+\psi(1+\lambda-\mu)\Big] \qquad \ldots \quad (11b)$$

where

$$\psi(z) = \frac{d}{dz}\log\Gamma(z)$$

Using (10) and (11) and the relation (1946)

$$\psi(1-z) = \psi(z)+\pi\cot\pi z$$

we get from (9)

$$\left.\frac{dS_0(k)}{dk}\right|_{k=0} = -2iR-4\gamma a-2ia\cdot Q \qquad . \quad (12)$$

where

$$Q = [\psi(\lambda_0+\mu)+\psi(-\lambda_0+\mu)]+\pi$$

$$\frac{\left[\begin{array}{l}\left(\frac{b}{1+b}\right)^{\lambda_0}\left(F(\lambda_0+\mu, 1+\lambda_0-\mu, 1+2\lambda_0; \frac{b}{1+b}\right)\frac{\Gamma(1-2\lambda_0)}{\Gamma(1-\lambda_0-\mu)\,\Gamma(-\lambda_0+\mu)}\cdot\cot\{\pi(\lambda_0+\mu)\}\\ -\left(\frac{b}{1+b}\right)^{-\lambda_0}\left(F(-\lambda_0+\mu, 1-\lambda_0-\mu, 1-2\lambda_0; \frac{b}{1+b}\right)\frac{\Gamma(1+2\lambda_0)}{\Gamma(1+\lambda_0-\mu)\Gamma(\lambda_0+\mu)}\cot\{\pi(-\lambda_0+\mu)\}\end{array}\right]}{\left[\begin{array}{l}\left(\frac{b}{1+b}\right)^{\lambda_0}\left(F(\lambda_0+\mu, 1+\lambda_0-\mu, 1+2\lambda_0, \frac{b}{1+b}\right)\frac{\Gamma(1-2\lambda_0)}{\Gamma(1-\lambda_0-\mu)\,\Gamma(-\lambda_0+\mu)}\\ -\left(\frac{b}{1+b}\right)^{-\lambda_0}\left(F(-\lambda_0+\mu, 1-\lambda_0-\mu, 1-2\lambda_0; \frac{b}{1+b}\right)\frac{\Gamma(1+2\lambda_0)}{\Gamma(1+\lambda_0-\mu)\,\Gamma(\lambda_0+\mu)}\end{array}\right]}$$

γ = Euler's constant and λ_0 is the value of λ at $k = 0$. Our expression is symmetric with respect to $\pm\lambda$ and μ and $1-\mu$ Separating (12) into real and imaginary parts we get the expressions $\frac{\Gamma_n^0}{D}$ and R' corresponding to a nuclear potential with both volume and surface absorption as follows

$$\frac{\Gamma_0^n}{D} = -\frac{2k_0a}{\pi}\,\mathrm{Im}\,(Q) \qquad \ldots \quad (13)$$

$$R' = R+2\gamma a+a\,\mathrm{Re}(Q) \qquad \ldots\ldots \quad (14)$$

For Woods-Saxon potential without surface absorption we put $\mu = 0$ in (12). Further taking the limit $a \to 0$, we have for the square well case

$$\frac{dS_0(k)}{dk}\Big|_{k=0} = -2iR + \frac{2i}{p}\tan pR \tag{15}$$

where
$$p = \left[\frac{2m}{\hbar^2}(V_0 + iW_0)\right]^{\frac{1}{2}} \tag{16}$$

$$\frac{\Gamma_n^0}{D} = \frac{2K_0R}{\pi} \cdot \frac{X_1 \sinh 2X_2 - X_2 \sin 2X_1}{(\cos 2X_1 + \cosh 2X_2)(X^2_1 + X^2_2)} \quad \ldots \tag{17}$$

where
$$pR = X_1 + iX_2$$

The expression (17) for $\frac{\Gamma_n^0}{D}$ is identical with the analytic form given by Feshbach *et al* (1954). The detailed comparison of theoretical and experimental results is in progress.

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